II. Solution by A. M. HARDING, University of Arkansas, and JAMES A. BULLARD, Worcester Polytechnic Institute.

Let $f(x,\ y) \equiv y^3-3y+x=0$. Then

$$\frac{dy}{dx}=-\frac{\partial f/\partial x}{\partial f/\partial y}=-\frac{1}{3y^2-3}=f'(x,\ y);\ \frac{d^2y}{dx^2}=\frac{-6y}{(3y^2-3)^3}=f''(x,\ y);$$

$$\frac{d^3y}{dx^3}=\frac{-6(15y^2+3)}{(3y^2-3)^5}=f'''(x,\ y);\ \frac{d^4y}{dx^4}=\frac{-24.45y(2y^2+1)}{(3y^2-3)^7}=f^{\mathrm{iv}}(x,\ y);$$

$$\frac{d^5y}{dx^5}=\frac{-120.9(56y^4+57y^2+3)}{(3y^2-3)^9}=f^{\mathrm{v}}(x,\ y);\ \ldots$$

Substituting the values of these derivatives for $x=0$, *i. e.* when $y=0$, in Maclaurin's formula, we have

$$y=f(0)+f'(0)x+\frac{f''(0)}{2!}x^2+\frac{f'''(0)}{3!}x^3+\frac{f^{\mathrm{iv}}(0)}{4!}x^4+\frac{f^{\mathrm{v}}(0)}{5!}x^5+\ldots$$

$$=\frac{x}{3}+\frac{x^3}{3^4}+\frac{x^6}{3^6}+\ldots$$

NOTE. When $x=0$, $y=0$, $\sqrt{3}$, or $-\sqrt{3}$. Then there are two other series for y obtained by substituting $y=\sqrt{3}$ and $y=-\sqrt{3}$ in the above derivations.

Solved similarly by V. M. Spunar, S. G. Barton, M. E. Graber, and C. N. Schmall.

313. Proposed by M. E. GRABER, A. M., Heidelberg University, Tiffin, Ohio.

Evaluate the definite integral $\int_0^\infty (e^{-2ax^2}+e^{2ax^2})\,dx$.

Solution by C. N. SCHMALL, New York City.

$$\int_0^\infty (e^{-2ax^2}+e^{2ax^2})\,dx=\int_0^\infty e^{-2ax^2}dx+\int_0^\infty e^{2ax^2}dx.$$

The first of these two integrals can be evaluated by meams of the familiar result $\int_0^\infty e^{-x^2}dx=\frac{1}{2}\sqrt{\pi}$ (see Williamson's *Integral Calculus*).

Putting $x\sqrt{(2a)}$ for x, we have

$$\int_0^\infty e^{-2ax^2}dx=\tfrac{1}{2}\sqrt{\frac{\pi}{2a}}.$$

Now, the expression e^{2ax^2} can be integrated only, so far as we know, by expanding it in an infinite series, thus:

$$e^{2ax^2}=1+2ax^2+\frac{4a^2x^4}{1.2}+\dots.$$

$$\therefore \int e^{2ax^2}dx=x+\frac{2ax^3}{1.3}+\frac{4a^2x^5}{1.2.5}+\dots.$$

NOTE. Professors Harding and Prime should have received credit for solving 308.

PROBLEMS FOR SOLUTION.

ALGEBRA.

366. Proposed by WILLIAM HOOVER, Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

Eliminate m from the equations

$$3a^2m^4+4aym^3+6axm^2-(x^2+y^2-4ax)=0;$$
$$(x^2+y^2-4ax)m^4-6axm^2+4aym-3a^2=0.$$

367. Proposed by E. B. ESCOTT, University of Michigan, Ann Arbor, Michigan.

Solve the simultaneous equations:

$$\frac{2x}{1+x^2}=y\dots(1);\quad \frac{2y}{1+y^2}=z\dots(2);\quad \frac{2z}{1+z^2}=u\dots(3);\quad \frac{2u}{1+u^2}=x\dots(4).$$

368. Proposed by E. B. ESCOTT, University of Michigan, Ann Arbor, Michigan.

Solve the functional equation, $\dfrac{f(-x)}{f(x)}=r^{2x}$.

GEOMETRY.

399. Proposed by J. K. ELLWOOD, Superintendent of Schools, Lucas, Kansas.

A race track is to be composed of two tangents and the arc of the circle which is concave towards the point of intersection of the two tangents, each tangent and the arc of the circle being 1 mile. What is the radius of the circle?

400. Proposed by FRANCIS RUST, C. E., Pittsburgh, Pennsylvania.

Given a circle and a point P without; construct, *using the straight edge only*, the two tangents to the circle through P.